Rossmoyne Senior High School

Semester Two Examination, 2022

Question/Answer booklet

MATHEMATICS APPLICATIONS UNITS 3&4

Section Two:
Calculator-assumed

SOLUTIONS

WA student number: In figures

In words

Your name

Number of additional answer booklets used (if applicable):

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material with you. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	51	35
Section Two: Calculator-assumed	12	12	100	99	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **65% (99 Marks)**

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8 **(6 marks)**

During the spring of 2020, a national real estate company observed that from a sample of 300 households in Adelaide who rented their accommodation and applied to their landlord for a rent reduction, 201 applications were approved. From a similar survey in Sydney of 450 households, 261 applications for a rent reduction were approved.

(a) Use the above information to complete the two-way frequency table below. (2 marks)

Rent Reduction / City	Adelaide	Sydney
Approved	**201**	**261**
Not approved	**99**	**189**
Total	**300**	**450**

Solution
See table
Specific behaviours
✓ one correct column
✓ correct table

The real estate company wanted to know whether the data provided any clear evidence of the presence of an association between rent reduction approvals and city.

(b) Complete the column percentaged two-way frequency table below. (2 marks)

Percentages	Adelaide	Sydney
Approved	**67**	**58**
Not approved	**33**	**42**
Total	**100**	**100**

Solution
See table
Specific behaviours
✓ one correct column
✓ correct table

(c) State, with justification, whether the data provides clear evidence of the presence of an association between rent reduction approvals and city. (2 marks)

Solution
There is clear evidence of an association, as the percentage approval rates for the cities differ by 9%.
Specific behaviours
✓ state data does provide clear evidence
✓ justifies by referring to different percentages along rows

Question 9 **(6 marks)**

An injection moulding machine was purchased by a business to make plastic chairs. The initial value of the machine was $7500, and this value depreciates at a constant rate of 8 cents per chair made.

(a) Calculate the loss in value of the machine after 2000 chairs have been made. (1 mark)

Solution
$2000 \times 0.08 = \$160.$
The machine has lost $160 in value.
Specific behaviours
✓ correct loss in value

The value of the machine, in dollars, after n chairs have been made is given by $T_n = a - bn$.

(b) State the value of the constant a and the value of the constant b. (2 marks)

Solution
$T_n = 7500 - 0.08n$
Hence $a = 7500$ and $b = 0.08$.
Specific behaviours
✓ value of a
✓ value of b

(c) Calculate the value of the machine after 33 550 chairs have been made. (1 mark)

Solution
$T_{33\,550} = \$4816.$
Specific behaviours
✓ correct value

(d) The machine will be scrapped once its value falls to $700. Determine the number of chairs that the machine must make to reach this value. (2 marks)

Solution
$7500 - 0.08n = 700$
$n = 85\,000$
It must produce 85 000 chairs.
Specific behaviours
✓ correctly forms equation
✓ correct number of chairs

Question 10 **(6 marks)**

The number of tumble driers sold each month at a store are shown in the table below.

Month	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov
Number	29	35	41	50	55	58	65	57	40	–

(a) Determine

(i) the three-point moving average for August. (1 mark)

Solution
$(58 + 65 + 57) \div 3 = 60$
Specific behaviours
✓ correct value

(ii) the six-point moving average centred on June. (2 marks)

Solution
$\left(\dfrac{35}{2} + 41 + 50 + 55 + 58 + 65 + \dfrac{57}{2}\right) \div 6 = 52.5$
Specific behaviours
✓ indicates correct calculation
✓ evaluates correctly

(b) Explain the purpose of centring a moving average with an even number of data points.

(1 mark)

Solution
To align the moving average with time (i.e., the middle of the time period).
Specific behaviours
✓ reasonable explanation

(c) Determine the number of tumble driers sold in November, given that the four-point moving average for the data, centred on September, is 52. (2 marks)
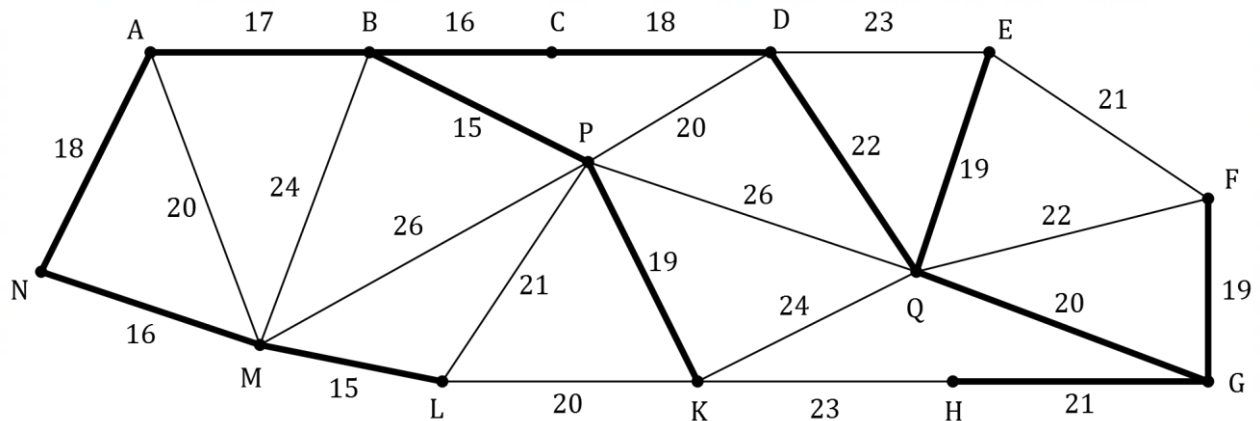
Solution
$\left(\dfrac{58}{2} + 65 + 57 + 40 + \dfrac{x}{2}\right) \div 4 = 52$
$x = 34$
Number sold was 34.
Specific behaviours
✓ indicates suitable method
✓ states correct value

Question 11 (7 marks)

The edges in the graph below represent power lines between 14 buildings, and the weight on each edge is the cost, in hundreds of dollars, to upgrade that line to carry more power.

(a) Clearly show the minimum spanning tree on the graph and hence determine the cost of upgrading all the power lines that form the minimum spanning tree. (5 marks)

Solution
LMNABPK: $15 + 16 + 18 + 17 + 15 + 19 = 100$.
BCDQE: $16 + 18 + 22 + 19 = 75$. QGH, GF: $21 + 20 + 19 = 60$.
Hence sum of edges in tree is 235 and so cost of upgrade is \$23 500.
Specific behaviours
✓ a spanning tree
✓ a spanning tree with at least 10 correct edges
✓ correct minimum spanning tree
✓ indicates correct sum of edges in tree
✓ correct cost of upgrade **in hundreds of dollars**

(b) Given that Prim's algorithm was used to determine the minimum spanning tree for the graph above, state the final edge that would be connected to complete the minimum spanning tree, when

(i) building M was the first vertex used with the algorithm. (1 mark)

Solution
Vertices **G** and **H**.
Specific behaviours
✓ correct pair of vertices

(ii) building E was the first vertex used with the algorithm. (1 mark)

Solution
Vertices **P** and **K**.
Specific behaviours
✓ correct pair of vertices

Question 12 **(8 marks)**

Annie is keen to buy a car and has been offered a reducing balance loan of $22 500 to help with her purchase. The loan is to be repaid in 24 equal monthly payments of $1036.60, and Annie has started the spreadsheet below to investigate how the balance of the loan reduces.

Month (n)	Opening balance	Interest	Repayment	Closing balance (T_n)
1	22 500.00	184.50	1036.60	21 647.90
2	21 647.90	177.51	1036.60	20 788.81
3	20 788.81			

(a) Use figures from the first month to deduce that the annual interest rate is 9.84%. (1 mark)

Solution
$184.50 \div 22\ 500 \times 12 = 0.0082 \times 12 = 0.0984$
Hence rate is 9.84% p.a.
Specific behaviours
✓ shows correct calculation(s) to obtain interest rate

(b) Determine the interest for the third month and hence state the closing balance for that month. (2 marks)

Solution
$I = 20\ 788.81 \times 0.0082 = \$170.47$
$T_3 = 20\ 788.81 + 170.47 - 1036.60$
$= \$19\ 922.68$
Specific behaviours
✓ correctly calculates interest
✓ correct closing balance

(c) Write a recurrence relation for T_n, the closing balance for month n. (2 marks)

Solution
$T_{n+1} = 1.0082T_n - 1036.60, \quad T_0 = 22\ 500$
$0r\ T_1 = 21647.90$
Specific behaviours
✓ states correct rule
✓ states correct initial term

(d) Determine the closing balance of the loan at the end of the twelfth month. (1 mark)

Solution
$T_{12} = \$11\ 800.80$
Specific behaviours
✓ correct closing balance

(e) Determine the total interest paid on the loan over the 24 months. (2 marks)

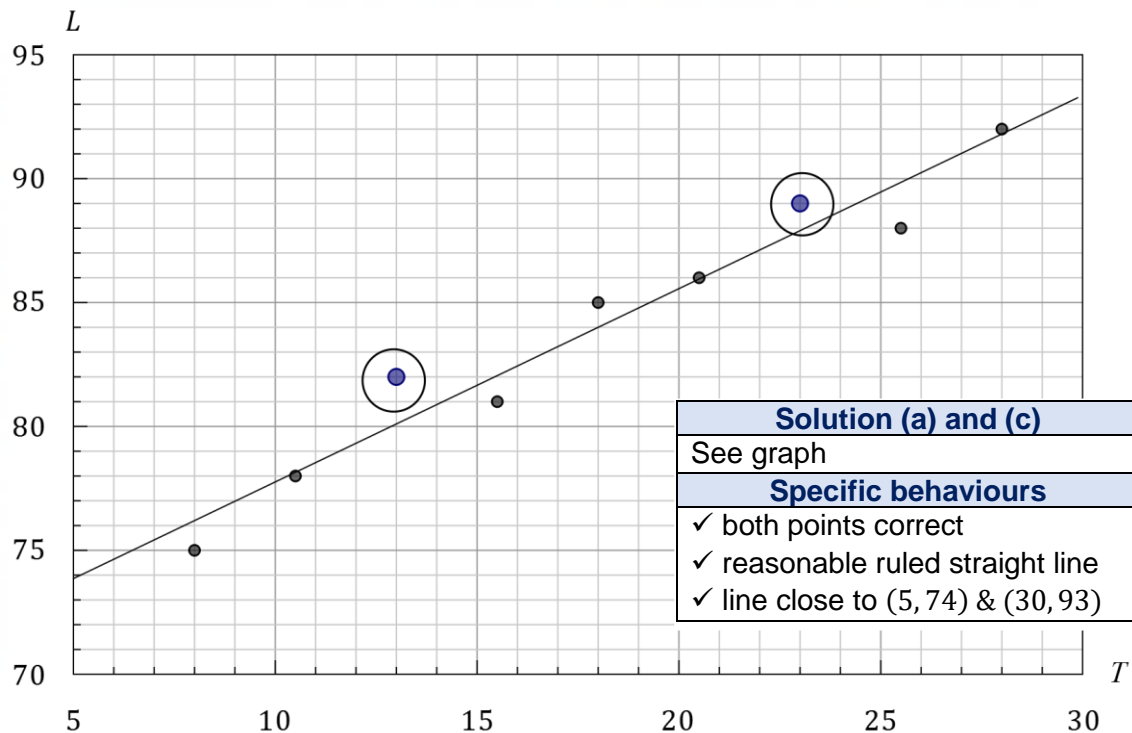| Solution |
|---|
| Repayments: $24 \times 1036.60 = 24\ 878.40$. |
| Hence total interest is $24\ 878.40 - 22\ 500 = \$2378.40$ |
| **Specific behaviours** |
| ✓ indicates correct method (possibly financial calculator) |
| ✓ correct total interest |

Question 13   **(12 marks)**

A researcher is analysing data to confirm Allen's Rule, a century-old biological observation that an association exists between ambient temperature and limb length in mammals. The table shows the tail length in millimetres of 8-week-old mice and the temperature in degrees Celsius that they were housed at since birth.

Temperature, T	8.0	10.5	12.0	15.5	18.0	20.5	23.0	25.5	28.0
Tail length, L	75	78	82	81	85	86	89	88	92

(a)   On the scatterplot below, plot and circle the two missing data points from the table.

(1 mark)

Solution (a) and (c)
See graph
Specific behaviours
✓ both points correct
✓ reasonable ruled straight line
✓ line close to $(5, 74)$ & $(30, 93)$

(b)   Determine the equation of the least-squares line for the data.   (2 marks)

Solution
$\hat{L} = 0.763T + 70.35$
Specific behaviours
✓ correctly determines coefficients (shown to at least 2 sf)
✓ correctly writes equation using given variables

(c)   Draw the least-squares line on the scatterplot above.   (2 marks)

(d)   Interpret the slope of the least-squares line in the context of this question.   (2 marks)

Solution
For every $1°C$ increase in temperature, the tail length increases by an average of 0.763 mm.
Specific behaviours
✓ refers to correct variables
✓ correctly states average length increase

(e)   Determine the value of the correlation coefficient between T and L, and use it to describe the association between the variables in terms of direction and strength.   (2 marks)

Solution
$r_{TL} = 0.968$
The association is positive and strong.
Specific behaviours
✓ correct coefficient (shown to at least 2 sf)
✓ correctly states direction and strength

(f)   Use the equation of the least-squares line to predict the tail length of a 8-week-old mouse that was housed at a temperature of $38°C$, and comment on the validity of this prediction.
(2 marks)

Solution
$L = 0.763(38) + 70.35$
$= 99.35 \text{ mm}$
This prediction is not valid as it involves extrapolation.
Specific behaviours
✓ correct prediction
✓ states not valid due to extrapolation

(g)   The researcher stated, "This data clearly shows that housing mice at higher temperatures causes their tails to grow longer". Comment on the validity of this statement.   (1 mark)

Solution
The statement is not valid since an observed association between the variables does not necessarily mean a causal relationship exists.
Specific behaviours
✓ indicates statement not valid, with appropriate reason

Question 14   **(11 marks)**

The number of enquiries received each quarter by a party hire business are shown in the table below, together with some derived figures.

Year	Quarter	n	Number of enquiries (E)	Seasonal mean (M)	$E \div M$ (%)
2014	1	1	407	365	111.51
	2	2	298		81.64
	3	3	A		90.96
	4	4	423		115.89
2015	1	5	497	450	110.44
	2	6	428		95.11
	3	7	356		79.11
	4	8	519		115.33
2016	1	9	591	B	C
	2	10	454		89.72
	3	11	405		80.04
	4	12	574		113.44

(a) Calculate the value of A, the value of B and the value of C in the table.   (3 marks)

Solution
$A = 4 \times 365 - 407 - 298 - 423 = 332$ or $A = 365 \times 0.9096 = 332$
$B = (591 + 454 + 405 + 574) \div 4 = 506$
$C = 591 \div 506 \times 100 = 116.80$
Specific behaviours
✓ value of A **(must be rounded)**
✓ value of B
✓ value of C **(must be 2dp)**

(b) Complete the table below, to show the seasonal index for each quarter.   (2 marks)

Quarter	1	2	3	4
Seasonal index	1.129	**0.888**	0.834	**1.149**

Solution
Quarter 2: $(0.8164 + 0.9511 + 0.8972) \div 3 = 0.888$
Quarter 4: $4 - 1.129 - 0.888 - 0.834 = 1.149$
Specific behaviours
✓ index for Q2
✓ index for Q4

(c)   Determine in which quarter of 2015 the deseasonalised number of enquiries was the lowest, and state what this deseasonalised number is.   (2 marks)

Solution
$497 \div 1.129 = 440$
$428 \div 0.888 = 482$
$356 \div 0.834 = 427$
$519 \div 1.149 = 452$
Hence lowest in third quarter, when it was 427.
Specific behaviours
✓ states correct quarter
✓ states correct value

The equation of the least-squares line for the deseasonalised number of enquiries (e) against time period (n) is $\hat{e} = 15.49n + 339.57$, and the correlation coefficient is 0.88.

(d)   Use the equation of the least-square line above and seasonal adjustments as required to predict the number of enquiries received by the business in the third quarter of 2017.   (2 marks)

Solution
$\hat{e} = 15.49(15) + 339.57 = 571.92,$ $571.92 \times 0.834 = 477.0.$
Hence predict 477 enquiries.
Specific behaviours
✓ correctly uses $n = 15$ in least squares line
✓ correctly multiplies by seasonal index and rounds to whole number

(e)   Time series predictions inevitably involve extrapolation. Ignoring this factor and assuming that a strong association exists, state two other assumptions required for a reasonable level of confidence in predictions such as that made in part (d).   (2 marks)

Solution
It must be assumed that:
1. The existing trend in the number of enquiries continues.
2. The existing seasonality in the number of enquiries continues.
Specific behaviours
✓ indicates trend continues
✓ indicates seasonality continues

Question 15   **(7 marks)**

The annual cost of electricity A_n for a household, in the nth year after 2017, is shown in the table below in dollars.

Year	2017	2018	2019
n	0	1	2
A_n	2100.00	2247.00	2404.29

(a)   Use the values of A_n in the table to demonstrate that the annual cost of electricity increased by 7% every year.   (2 marks)

Solution
$2247 \div 2100 = 1.07$, $2404.29 \div 2247 = 1.07$
Hence annual increase of 7%.
Specific behaviours
✓ uses two consecutive terms to obtain common ratio
✓ uses second pair of terms to confirm same ratio

For the remainder of this question, assume that the annual cost of electricity for the household continues to increase by 7% each year.

(b)   Determine the annual cost of electricity for the household in 2020.   (1 mark)

Solution
$2404.29 \times 1.07 = \$2572.59$
Specific behaviours
✓ correct cost, to nearest cent

(c)   Determine a rule for the nth term of A_n.   (2 marks)

Solution
$A_n = 2100(1.07)^n \text{ or } A_n = 2247(1.07)^{n-1}$
Specific behaviours
✓ uses correct coefficients in rule for geometric sequence

(d)   Determine which year e.g. 2023 the annual cost of electricity for the household first exceed \$4500, and state the cost in this year.   (2 marks)

Solution
$A_{12} = 4729.60$
When $n = 12$, the year will be 2029 and the cost will be \$4729.60.
Specific behaviours
✓ states correct year
✓ states correct cost

See next page

Question 16 **(7 marks)**

Eight students were asked to complete a physical task using both hands and the time taken recorded as t_1 seconds. The students then repeated the task using just one hand, and the new time taken recorded as t_2 seconds. The table below shows the paired data for these students.

t_1	17	16	11	12	24	26	22	20
t_2	37	34	32	34	39	41	37	39

The equation of the least squares line for this data is $\hat{t}_2 = 0.52t_1 + 27.0$, and the correlation coefficient is 0.93.

(a) Which is the explanatory variable? (1 mark)

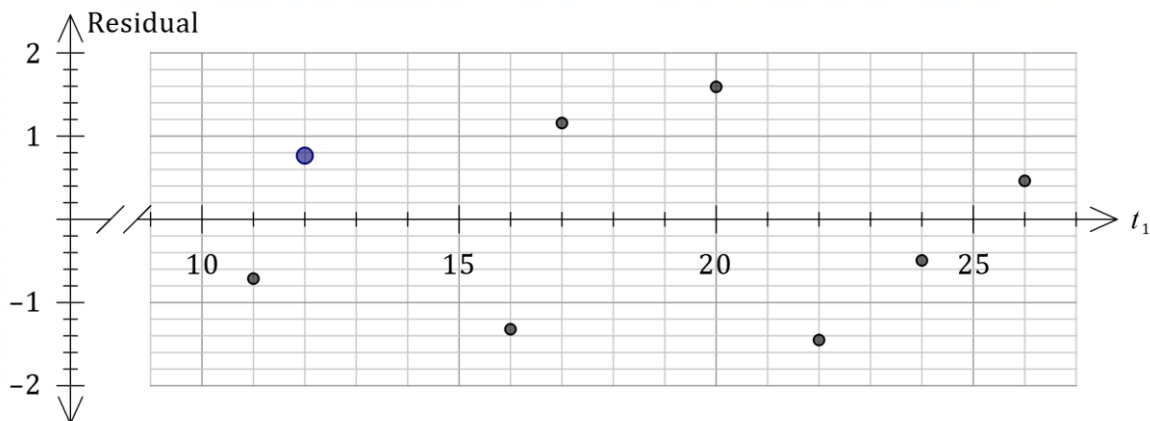| **Solution** |
|---|
| Time using both hands, t_1. |
| **Specific behaviours** |
| ✓ states t_1 |

(b) What percentage of the variation in t_2 can be explained by the variation in t_1? (1 mark)

Solution
$r^2 = 0.93^2 = 0.8649$, and so 86% of the variation can be explained in this way.
(***actually $r = 0.9269$ using CAS***)
Specific behaviours
✓ correct percentage that rounds to 86%

The residual plot for seven of the eight data points is shown below.

(c) Calculate the residual for the student with $t_1 = 12$ and add this point to the residual plot above. (3 marks)

Solution
$\hat{t}_2 = 0.52(12) + 27 = 33.24,$ $t_2 - \hat{t}_2 = 34 - 33.24 = 0.76$
(***actually $\hat{t}_2 = 33.2344$ using CAS***)
Specific behaviours
✓ calculates $\hat{t}_2$
✓ calculates residual
✓ correctly plots point

(d) Use the residual plot to justify whether the least-squares line is an appropriate model for the relationship between t_1 and t_2. (2 marks)

Solution
The least-squares line is appropriate as no pattern is evident in the residual plot.
Specific behaviours
✓ states model is appropriate
✓ justifies using no pattern evident in residual plot

Question 17 **(12 marks)**

Zac arranged a loan of \$85 000 with his bank so that for the first four years, no repayments were required. The loan attracted interest of 6.28% per annum, compounded quarterly.

(a) State a recurrence relation for A_n, the loan balance after n quarters. (2 marks)

Solution
$0.0628 \div 4 = 0.0157$
$A_{n+1} = 1.0157A_n, \qquad A_0 = 85\,000$
Specific behaviours
✓ indicates correct multiplier
✓ states recurrence relation with initial term

(b) The effective interest rate of the loan is 6.43% per annum. Write an expression that can be used to evaluate this rate from the advertised rate of 6.28% per annum. (1 mark)

Solution
$\left(1 + \dfrac{0.0628}{4}\right)^4 - 1 \ \text{ or } \ (1.0157)^4 - 1$
Specific behaviours
✓ correct expression

At the end of the four-year term, Zac repaid all the interest that had been added to the loan.

(c) Determine the total interest added to the loan during the first four years. (3 marks)

Solution
Loan balance after four years will be $A_{16} = 109\,060.17$.
$\begin{aligned} I &= A_{16} - A_0 \\ &= 109\,060.17 - 85\,000 \\ &= 24\,060.17 \end{aligned}$
Total interest is \$24 060.17.
Specific behaviours
✓ indicates 16 compounding periods or determines A_{16}
✓ calculates loan balance
✓ correct total interest

After Zac repaid the interest, the loan reverted to a reducing balance loan, with the principal of $85 000 repaid in twelve equal quarterly instalments. The first repayment was made three months after the end of the initial four-year term.

(d)   Determine the amount of each repayment.   (2 marks)

Solution
Using financial calculator with payment date set to End and $N = 12, I\% = 6.28, PV = 85\,000, FV = 0, C/Y = 4, \ P/Y = 4$ then $PMT = -7826.82$.   Each repayment is $7826.82.
Specific behaviours
✓ indicates correct settings and values for financial solver   ✓ correctly solves for repayment

(e)   State the balance of the loan at the end of the fifth year, after four repayments have been made.   (2 marks)

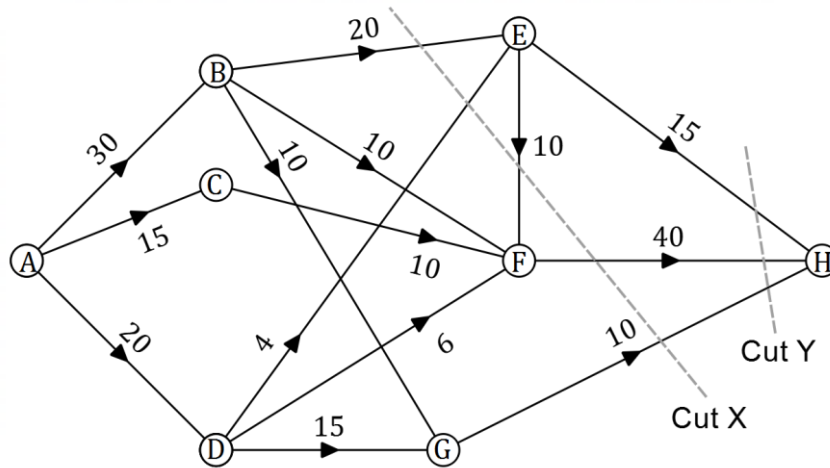| Solution |
|---|
| Using financial calculator with payment date set to End and $N = 4, I\% = 6.28, PV = 85\,000, PMT = -7826.82, C/Y = 4, \ P/Y = 4$ then $FV = -58\,412.72$.

 The balance will be $58\,412.72. |
| **Specific behaviours** |
| ✓ indicates correct settings and values for financial solver
 ✓ correctly solves for balance |

(f)   Determine the total interest added to the loan during the fifth year.   (2 marks)

Solution
Principal less four repayments is $85000 - 4 \times 7826.82 = 53\,692.72$.   Hence total interest added is $58\,412.72 - 53\,692.72 = \$4720.00$.
Specific behaviours
✓ indicates appropriate method (possibly using financial calculator)   ✓ correct total interest

Question 18 **(9 marks)**

The flow of air from intake A through a system of ducts to outlet H is shown in the network below. Each edge weight represents the maximum capacity of that duct, in cubic metres per minute.

(a) Determine the capacity of

(i) cut X. (1 mark)

Solution
Cut X: $20 + 4 + 0 + 40 + 10 = 74 \text{ m}^3/\text{min}$
Cut Y: $15 + 40 + 10 = 65 \text{ m}^3/\text{min}$

(ii) cut Y. (1 mark)

Specific behaviours
✓ correct capacity of cut X
✓ correct capacity of cut Y

(b) Is the maximum flow through the system of ducts from A to H equal to the largest of the capacities you found in part (a)? Justify your answer. (1 mark)

Solution
No, as the maximum flow cannot be more than the smaller cut of 65 cubic metres per minute.

Specific behaviours
✓ states no with reasonable justification

(c) State the maximum possible flow along the path ADEFH. (1 mark)

Solution
4 cubic metres per minute.

Specific behaviours
✓ correct flow

(d)   Determine the maximum flow through the system of ducts from A to H.   (3 marks)

Solution (listing of flows)
ABEH 15
ABEFH 5
ABFH 10
ACFH 10
ADEFH 4
ADFH 6
ADGH 10
Maximum flow 60 m^3/min.

Specific behaviours
✓ correct flow along at least one path
✓ correct flows along at least two more paths
✓ states correct maximum flow **with units**

Solution (cuts)
(Examples of cuts)
Cut though source: 65
Cut to right of BCD: 75
Cut to left of EFH: 60
Maximum flow 60 m^3/min.

Specific behaviours
✓ value of new cut (not X or Y)
✓ values of two more new cuts
✓ states correct maximum flow
with units

(e)   To achieve the maximum flow determined in part (d), not all ducts will be used to their maximum capacity. Clearly label all such ducts with their spare capacity on the copy of the network below.   (2 marks)
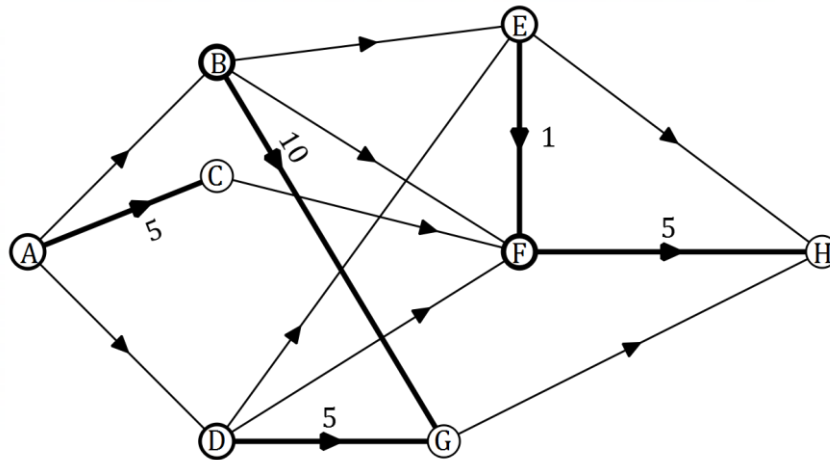
Solution
See diagram.
Note that EF[1] + FH[5] may be replaced with EH[1] + FH[4].

Specific behaviours
✓ correctly identifies and labels two or more ducts
✓ correctly identifies and labels all ducts

See next page

Question 19   **(8 marks)**

May currently has a nil balance in her savings account that earns 5.28% per annum, compounded monthly. She plans to fund a 12-month round-the-world trip by making deposits of $1250 into the account at the end of each month.

(a)   Determine the balance of her account just after her 12th deposit.   (2 marks)

Solution
$N = 12, I\% = 5.28, PV = 0, PMT = -1250, PY = 12, CY = 12 \rightarrow FV = \mathbf{15\,368.38}$
Balance will be $15 368.38.
Specific behaviours
✓ sets at least 5 correct values in financial calculator
✓ correct balance

As soon as her balance first exceeds $60 000, May will stop making deposits and head off on a 12 month trip. Because regular payments are no longer made into the account, the interest rate will decrease by 0.36% per annum. May will make the same withdrawal at the end of each month, so that after the 12th one her savings account balance will have fallen to $8000.

(b)   Determine the balance of her account when she stops making deposits.   (2 marks)

Solution
$I\% = 5.28, PV = 0, PMT = -1250, FV = 60\,000, PY = 12, CY = 12 \rightarrow N = \mathbf{43.6}$
$N = 44, I\% = 5.28, PV = 0, PMT = -1250, PY = 12, CY = 12 \rightarrow FV = \mathbf{60\,538.48}$
Balance will be $60 538.48.
Specific behaviours
✓ correct value of N
✓ correct balance

(c)   Determine the monthly withdrawal May will make on her trip.   (2 marks)

Solution
$N = 12, I\% = \mathbf{4.92}, PV = -60\,538.48, FV = 8000, PY = 12, CY = 12 \rightarrow PMT = \mathbf{4528.56}$
Withdrawal must be $4528.56.
Specific behaviours
✓ sets at least 5 correct values in financial calculator
✓ correct amount of withdrawal

(d)   Determine the total amount of interest that her savings account earned from the time May started saving for her trip until its end.   (2 marks)

Solution
Let I be the interest, so that $44 \times 1250 - 12 \times 4528.56 + I = 8000$.
$I = \$7342.72$
(*NB Using Amortization will result in* $5538.48 + 1804.25 = 7342.73$)
Specific behaviours
✓ indicates correct method
✓ correct total interest

End of questions

Supplementary page

Question number: _____